Physics Internal Assessment

In my project I researched a very serious problem that will affect humans in the future. This problem is the ever increasing amount of debris in space. As the amount of space debris increases, the possibility of collisions increases exponentially. In the future, space travel may become nearly impossible. By using a removal method that involves sending electrodynamic tethers into space, the debris could be successfully cleared. The tether creates electrical impulses in space, which cause the debris to move. This is able to occur through Michael Faraday's Law of Induction ("Faraday's Law of Induction," n.d.). The impulse causes the debris to gain velocity, thus gaining altitude. Most of the debris is in low-earth orbit, which is roughly 2,000 km above the Earth's surface (Brown, 2012). The final destination of the debris in this removal process is what is known as a Lagrange Point. This is a position in space where the net gravitational pull is effectively zero. The closest Lagrange Point, L_1, is approximately 1.5 million kilometers from Earth ("Lagrangian Point," 2016). I chose this as my destination. The majority of the space debris is smaller than 1 cm^3, and many of the particles are made of aluminum, so for the energy calculations, I used a 1 cm^3 particle of aluminum (Dunbar, 2015). The mass of the particle would be approximately 2.7 grams or .0027 kg ("Densities of Solids," n.d.). I wanted to find out how the time, velocity, radius, circumference, and many of the other factors that influence the orbits of the space debris are related to each other and how they change. I wanted to find, through the use of models and data analysis, if there were any constants that were present, but not already included in the general physics formulas for orbits. My specific research question was how much energy and power would it take to

move the .0027 kg piece of aluminum to the L_1 Lagrange Point from 2,000 km above the Earth's surface. What were the initial and final velocities and angular velocities of the debris? What was the relationship between time and radius? Overall, I had many questions about various relationships, but my general objective was to determine the value of above variables. I wanted to do this project because I am absolutely fascinated and inspired by space and space travel. I want space travel to have a future and space debris is one of the major obstacles standing in front of that goal. I have an understanding of the feasibility and the physics behind the tethers already. What I wanted to figure out was how long it would take and the various statistics associated with the actual orbits, not the physics inside the tethers themselves. I believed that if I could calculate and model this information, then humans would be one step closer to solving this daunting problem. I felt a personal connection because of my fascination.

I had three models/designs, I had a 1,000 orbit pathway, a 500 orbit pathway, and a 25 orbit pathway. For each model, the starting altitude or the radius of the orbit was 8,370,000 meters. This is because the space debris in low-earth orbit is 2,000,000 meters above Earth's surface and the distance to the center of the earth is approximately 6,370,000 meters (Cain, n.d.). The formulas for orbits, such as $v_{orbit} = \sqrt{\frac{GM}{r}}$, require that the r or radius be measured from the center of the two objects. The final radius for the three models was 1,506,370,000 meters because L_1 is 1.5 million km from Earth's surface. However, for each model the rate at which the orbits increased in their radii was different for the three orbit pathways. The 1,000 orbit model increased

by 1,498,000 meters per orbit, the 500 orbit model increased by 2,996,000 meters per orbit, and the 25 orbit model increased by 59,920,000 meters per orbit.

Explanation of Calculation Process For all three models:

Firstly, I recorded the radii for each orbit. Next, I multiplied the radii by 2π to calculate the circumference of the orbit. To calculate the velocity of the orbiting debris, I multiplied the gravitational constant, G, and the mass of the Earth, M, and divided the product by the radius of the orbit. I then took the square root of this quotient to attain the velocity of the orbiting the debris. To calculate the time of the individual orbits, I calculated the quotient of the circumference divided by the velocity for that orbit. To calculate the acceleration, I squared the velocity and divided the product by the radius. To calculate the angular velocity, I divided the velocity by the radius. To calculate the total mechanical energy or work, I multiplied the mass, .0027 kg, by -0.5 and by the velocity squared. To calculate the power I subtracted the work done in the final orbit by the work done in the first orbit. I divided the difference by the total time taken to move the space debris. The quotient was in the unit of a watt. Finally, to calculate the total time taken to complete the entire movement, I divided the number of seconds by 60 to attain the number of minutes. I then divided it by 60 to attain the number of hours. I divided it by 24 to attain days and then again by 365 to attain the number of years taken.

Just to make sure that I did my calculations correctly, I recalculated the time taken in each orbit by calculating $\frac{2\pi r}{v}$ *and I also computed* $2\pi\sqrt{\frac{r}{a}}$ to get the same answer each time. I recalculated the acceleration by multiplying the gravitational constant by the Earth's mass and then dividing that product by the radius squared. The final calculation I

did was that I tried to find a certain constant for certain relationship that were not already written in the formulas. One constant I did find without any prior research was for the ratio of $\frac{T^2}{R^3}$ when using the mass of the Earth, which always came out to be approximately $9.91 \cdot 10^{-14} s^2 m^{-3}$. I later researched this ratio to find out that Kepler's Third Law stated the very same fact, except Kepler's constant used the sun's mass so it came out to be 2.977 x 10^{-19} $s^2 m^{-3}$ for the objects that orbit the sun. The formulas used are listed/derived below.

$$Circumference = 2\pi r$$

$$Velocity_{orbit} = \sqrt{\frac{GM}{r}}$$

$$Time_{orbit} = \frac{Circumference}{Velocity_{orbit}}$$

$$Time_{orbit} = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r}{a}}$$

$$Acceleration = \frac{Velocity_{orbit}^2}{r} = \frac{GM}{r^2}$$

$$Angular\ Velocity = \omega = \frac{Velocity_{orbit}}{r}$$

In circular motion the net force = the force of gravity, so:

$$\sum F = \frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$mv^2 r^2 = rGMm$$

$$mv^2 r = GMm$$

$$mv^2 = \frac{GMm}{r}$$

$$Total\ Mechanical\ Enegy = Work = Potential + Kinetic = \frac{-GMm}{r} + \frac{1}{2}mv^2$$

From above, I can replace:

$$Work = -mv^2 + \frac{1}{2}mv^2 = -\frac{1}{2}mv^2$$

$$Power = \frac{\Delta Work}{\Delta Time}$$

(Broholm, 1997)

Data:

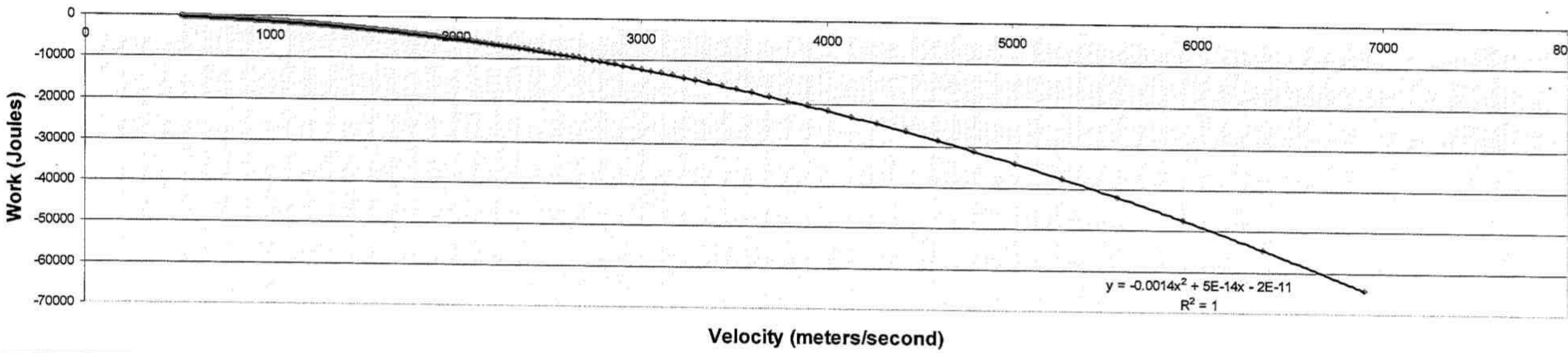
Number of Orbits	1000	500	25
Time in Years to complete	234.9858195	117.63894	6.1618706
Delta Work (Joules)	63926.96047	63926.96047	63926.96047
Power (Watts)	8.62652E-06	1.72316E-05	8.61581E-07
Initial Velocity (meters/second)	6900.57159	52590261.02	7621.145631
Final Velocity (meters/second)	514.377503	9464801851	18400497.29
Initial Acceleration (meters/second ^{2})	5.689114488	5.689114488	5.689114488
Final Acceleration (meters/second ^{2})	0.000175644	0.000175644	0.000175644
Initial Angular Velocity (1/second)	0.000824441	0.000824441	0.000824441
Final Angular Velocity (1/second)	3.41468E-07	3.41468E-07	3.41468E-07

The above data I generated myself using the said formulas, but the altitudes that the formulas are based on came from (Brown, 2012).

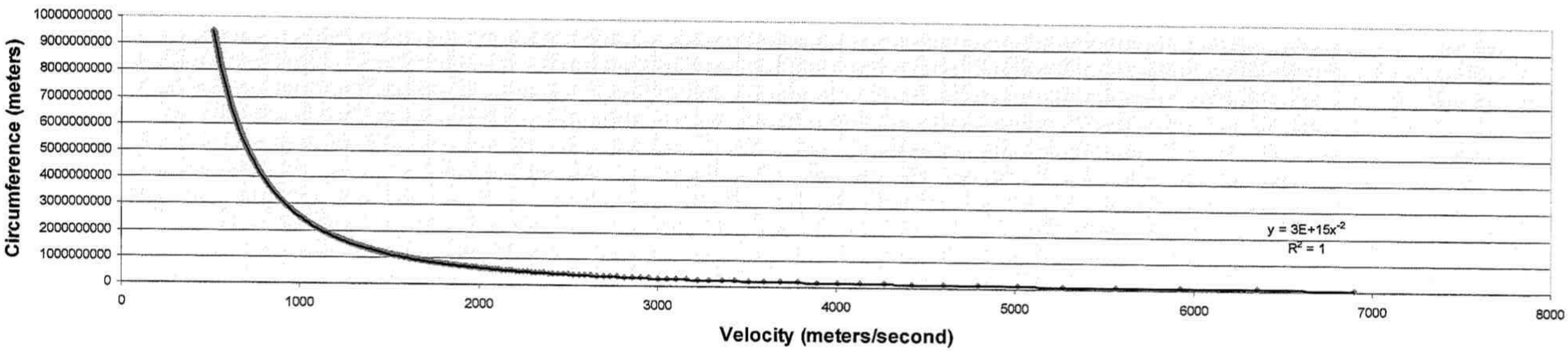
The unit for the angular velocity is usually radians per second, but I left it as s^{-1} because it could be degrees per second or another unit as the numerator.

All of the following graphs were based on the data from the 1,000 orbit model in order to give the most accurate line. The 1,000 orbit model is the most similar to the spiral orbit in real life because the change in altitude or the "jump" is the smallest value for the 1,000 orbit model.

Work/Velocity (kg*m/s or N*s)

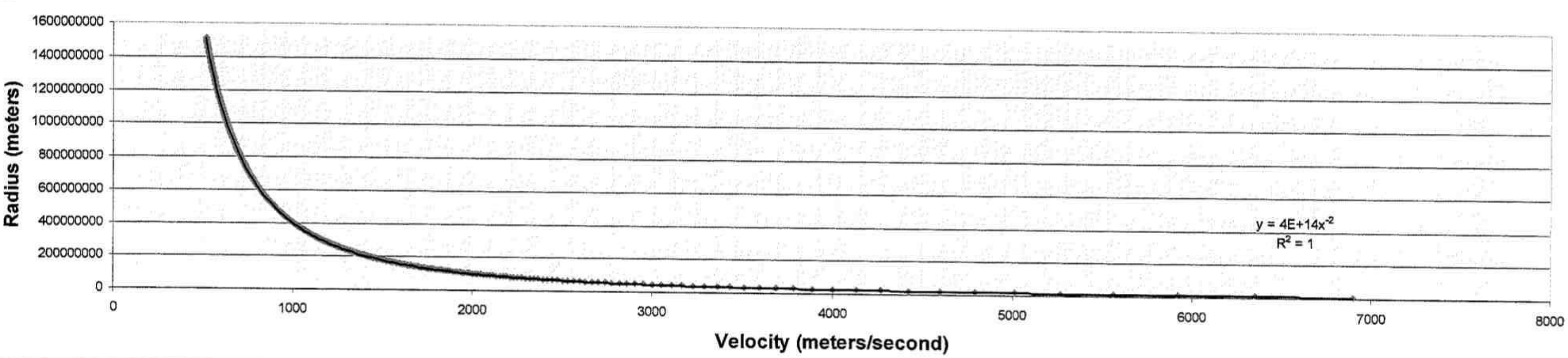


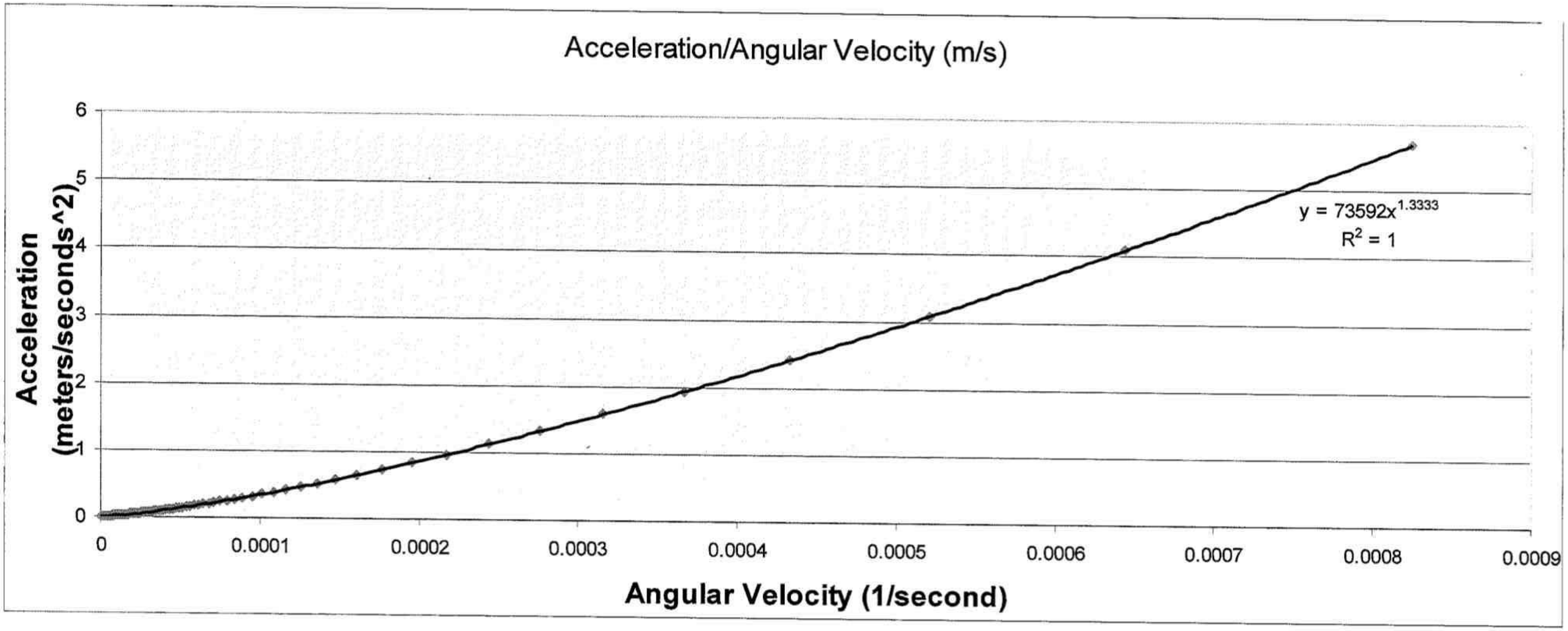
Circumference/Velocity (s)

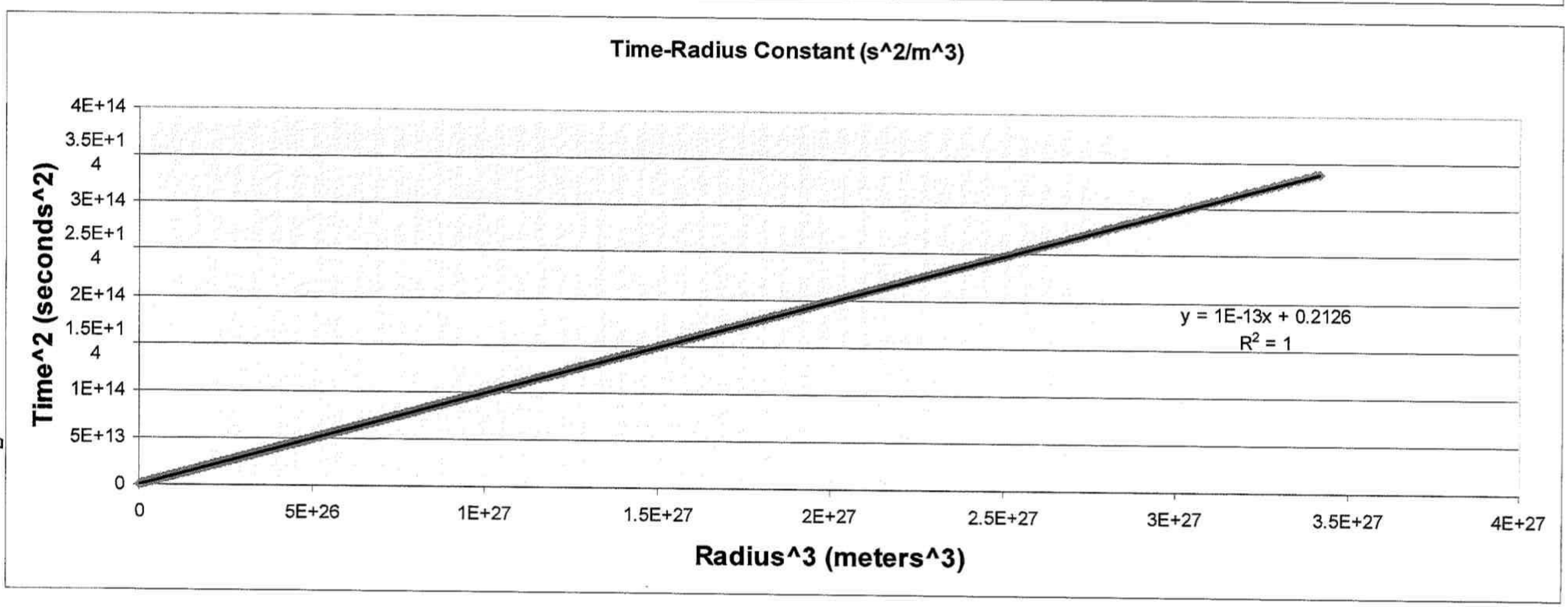


Duran

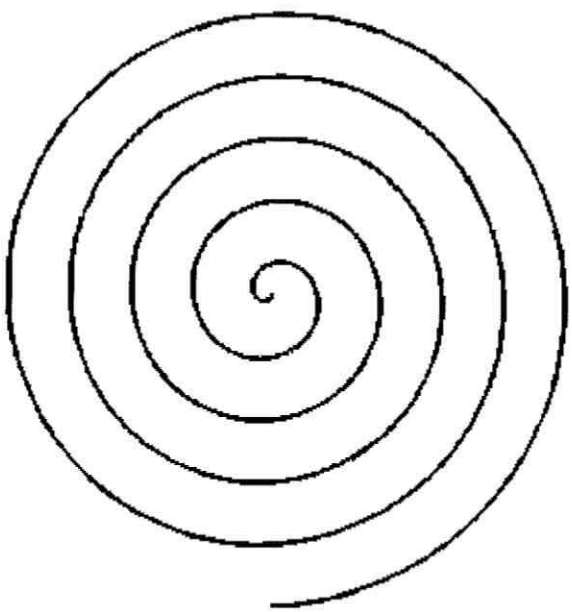
Radius/Velocity (s)



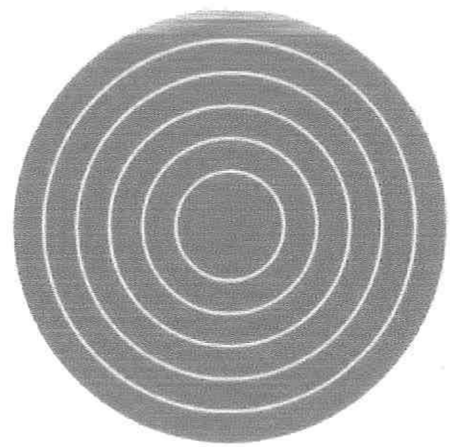

Acceleration/Angular Velocity (m/s)
y = 73592x^1.3333
R^2 = 1
Acceleration (meters/seconds^2)
Angular Velocity (1/second)
0
1
2
3
4
5
6
0
0.0001
0.0002
0.0003
0.0004
0.0005
0.0006
0.0007
0.0008
0.0009


Time-Radius Constant (s^2/m^3)
y = 1E-13x + 0.2126
R^2 = 1
Time^2 (seconds^2)
Radius^3 (meters^3)
0
5E+13
1E+14
1.5E+14
2E+14
2.5E+14
3E+14
3.5E+14
4E+14
0
5E+26
1E+27
1.5E+27
2E+27
2.5E+27
3E+27
3.5E+27
4E+27

The following pictures are given so that my model may be distinguishable between what the space debris movement would look like in reality. On the left, the spiral pathway is what the debris would follow in real life. On the right, the picture describes the way my model works. Instead of being a continuous path, the debris would "jump" from one orbit to another, once it has completed the prior orbit. This "jump" would not occur in real life, but for the sake of practicability for modeling this situation, I am hypothetically assuming that it would occur.



("MATLAB Cookbook," n.d.)



("Create Concentric Circles in PowerPoint," n.d.)

<u>Analysis/Results:</u>

If I were to account for the systematic error as described above, then the total time take would be less because the total distance traveled by the debris would be less. The

gain of altitude would occur much more gradually, so the work would be distributed more evenly over time, making the power required more realistic/meaningful. If I were to account for the systematic error of ignoring the gravitational force of other objects in space, such as the moon or the sun, then instead of being a perfect spiral pathway/orbit, the space debris would instead take a more elliptical orbit due to the effect of the unevenness of the gravitational force when the debris is moving through multiple gravity fields at the same time (Earth, moon, sun, etc.). The elliptical orbit would cause the velocities not only to differ at different altitudes or radii, but at different positions in the individual orbits. Basically, all of the calculations I made would be shifted in many different ways because of the fact that so many variables would influence the path of the debris in real life. It is quite hard to tell how those variables would effect each specific measurement (velocity, acceleration, etc.). Due to that complexity, I instead simplified the situation by accounting for only two bodies. From my data, the fewer orbits that you have, or the larger the jumps between the orbits are, the less time it takes to complete the process. The fewer orbits that you have, the more power it takes to move the debris, which is due to the decrease in time, since $\Delta Work$ stays the same. It would take 63,927 Joules of energy to complete the process for the .0027 kg sample of aluminum. It would require 8.62652E-06 Watts at 1,000 orbits, 1.72316E-05 Watts at 500 orbits, and 8.61581E-07 Watts at 25 orbits. For all three models, since the starting radii and the final radii were the same, the initial and final velocities and angular velocities were the same for each. The initial velocity was 6,901 m/s and the final velocity was 514 m/s. The initial angular velocity was 5.69 s^{-1} and the final angular velocity was 0.000176 s^{-1}. The

relationship between time and radius when I used the mass of the Earth as the mass that the object was orbiting was $9.91 \bullet 10^{-14} s^2 m^{-3}$. That is the ratio of $\frac{T^2}{R^3}$ when orbiting Earth.

Conclusion:

Overall, the fewer the number of orbits, the more power you need the electrodynamic tether to use. Also, the fewer orbits you have the quicker the process is. My recommendation to whomever wants to clean up the debris in real life is to make the spiral gradient as large as possible because when we have the resources to travel into space easily, we don't want to have to wait two hundred years (amount of time for the 1,000 orbit model). To keep this in perspective, even though my model seems realistic except for those two limitations (spiral and gravity from other objects), don't be fooled by the required amount of energy. It is not 63,927 Joules of energy and then we are done cleaning up. No, that is the amount of energy required for just one piece of aluminum weighing only 2.7 grams. It is going to take a lot more energy to solve this problem. Overall, I learned a lot from this IA. I never knew about Kepler's Third Law and I realized how bad this problem really was. In conclusion, the answer to my main question was for an object weighing .0027 kg, it will take 63,927 Joules and

$8.63 \bullet 10^{-6} \textit{Watts for } 1{,}000 \textit{ orbits}, 1.72 \bullet 10^{-5} \textit{ Watts for } 500 \textit{ orbits, and } 8.62 \bullet 10^{-7} \textit{Watts for } 25 \textit{ orbits.}$

With regard to the design of my model, it was flawed in the sense that I did successive circular orbits from the initial position to the final, when in reality the orbit would be one long spiral. The problem with calculations regarding the spiral is that I would require a knowledge of calculus which I have not been taught yet. So, for the sake of practicability, I used successive circular orbits for the space debris to travel through.

The debris would hypothetically jump from one orbit to the next higher one after completing the orbit that it was already on. This systematic error in my model probably caused an increase in my values for total distance traveled for each orbit, time, velocity, energy, and other various components. A hypothetical situation to describe the error follows: Circle 1 has point A on it. Circle two has point B on it. They both have center O. A straight line can be drawn through O, A, and B. Circle two is larger than circle 1. You find the circumference of both circles and add them. This sum is greater than the length of a spiral going from A to B, assuming the spiral goes around once. So, the distance to travel one complete circular orbit and a second complete circular orbit, which is larger than the first, is much greater compared to the distance needed to travel in a spiral from the same starting altitude to the same ending altitude. The only other variable that I did not account for in my model was the gravitational force of bodies other than the Earth and the space debris. I did not account for the moon or the sun due to the limitations of the formulas. The formulas account for only two bodies, and the formulas that account for a larger number of bodies require a background knowledge of calculus yet again. However, other than these two drawbacks, my model is quite useful in determining the various relationships and values.

Finally, I will spend the rest of my life researching this topic so that I may finally solve the problem, but I will model it to the real world. I can extend this paper by accounting for the gravitational pull of other objects in space and by accounting for the spiral/elliptical pathway by using calculus, which I will learn in the future. I will find the realistic answer and from there I will spend the rest of my life trying to clean space of debris.

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International Baccalaureate
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For first use in 2016 assessment

4/ICCS

Group 4: Individual candidate cover sheet (biology, chemistry and physics)

Arrival date: **20 April / 20 October**

Session: May 2016

School number:

School name:

- *Complete this form in the working language of your school (English, French, Spanish).*
- *The form must be completed by the teacher and candidate.*
- *A completed copy should be retained by the school.*

Subject:

Level: SL

Candidate name:

Session number:

Candidate section:

To be completed by the candidate.

Title of the group 4 project: Geothermal Energy

Write a reflective statement of about 50 words outlining your involvement in the group 4 project:

I learned about the roles that conduction, convection, and radiation play in creating geothermal energy. Working as a team was enjoyable because I collaborated with very responsible and trustworthy students. I felt like I was a leader.

Title of individual investigation: Space Debris: Is there a solution?